

Quantum Langevin equations for semiconductor light-emitting devices and the photon statistics at a low-injection level

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From the microscopic quantum Langevin equations (QLEs) we derive the effective semiconductor QLEs and the associated noise correlations which are valid at a low-injection level and in real devices. Applying the semiconductor QLEs to semiconductor light-emitting devices (LEDs), we obtain a new formula for the Fano factor of photons which gives the photon-number statistics as a function of the pump statistics and several parameters of LEDs. Key ingredients are non-radiative processes, carrier-number dependence of the radiative and non-radiative lifetimes, and multimodeness of LEDs. The formula is applicable to the actual cases where the quantum efficiency η differs from the differential quantum efficiency η_d , whereas previous theories implicitly assumed $\eta = \eta_d$. It is also applicable to the cases when photons in each mode of the cavity are emitted and/or detected inhomogeneously. When $\eta_d < \eta$ at a running point, in particular, our formula predicts that even a Poissonian pump can produce sub-Poissonian light. This mechanism for generation of sub-Poissonian light is completely different from those of previous theories, which assumed sub-Poissonian statistics for the current injected into the active layers of LEDs. Our results agree with recent experiments. We also discuss frequency dependence of the photon statistics.

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I. INTRODUCTION

There have been many researches on quantum noise in light-emitting devices (LEDs) since the celebrated work of Shawlow and Townes [1]. We here consider the quantum noise in semiconductor LEDs. This subject was first studied by Haug and Haken [2], who derived from a microscopic model useful formulae for the first and second optical coherence of LEDs.

Recently, much attention has been paid to sub-Poissonian light (SPL), which has the intensity fluctuations lower than the standard quantum limit [3], of various LEDs [4–14]. The mechanism for generation of SPL in LEDs is quite different from that of squeezed light in nonlinear crystals [15]. The latter mechanism is well understood as a Bogoliubov transformation of a coherent state into a squeezed state of light [16]. The former mechanism, on the other hand, is often described by the Langevin theory of laser [3,17–21]. Previous studies on SPL in LEDs assumed that only a single mode or a few modes of photons are excited [4,6,14]. When the injection level is lowered, however, many modes of photons become

relevant, and we must consider all of them. Simple theories for such a case were reported in Refs. [5,7]. However, they are too simplified so that they cannot explain recent experiments by Hirano, Kuga (HK) and coworkers [9–11], who demonstrated that the experimental results at a low-injection level (LIL) disagree with the predictions of the simplified theories. The disagreement appears when the quantum efficiency η differs from the differential quantum efficiency η_d . HK [9] suggested that non-radiative processes might be responsible for the disagreement.

In this paper, to resolve the discrepancy between the previous theories and the experiments, we theoretically investigate the quantum noise in LEDs *at a low-injection level* [22]. To this end, we extend the Langevin-equation method of Chow, Koch and Sargent [21] to treat the case where many photon modes are excited in an LED. From the microscopic quantum Langevin equations (QLEs) and the associated noise correlations, we derive the semiconductor QLEs and the noise correlations at the LIL. An important assumption is that the photon-absorption and emission rates are much smaller than the photon-escape rate from a “cavity”.¹ In the experiment [12], it has been reported that $\eta_d/\eta > 2$ in low-injection regions, whereas $\eta \simeq \eta_d$ in high-injection regions. The difference between η and η_d is, in our theory, attributed to non-radiative processes and carrier-number dependence of lifetimes. They always exist in real LEDs, and become particularly important in low-injection regions. In contrast, we can easily show that the previous simplified theories [4–7] always give $\eta = \eta_d$. Hence our theory is a minimal one to simulate real LEDs. A new formula for the photon Fano factor [6] is derived. It gives the photon-number statistics as a function of the pump statistics, measuring frequency, η , η_d and some factors arising from *multimodeness* of LEDs. Simplified formulae are derived for homogeneous cases, in which photons in each mode of the cavity are emitted and detected homogeneously, and for inhomogeneous cases, in which they are emitted and/or detected inhomogeneously. Using these formulae, we discuss the condition for generation of SPL in an LED. Our results agree with the experimental results.

The paper is organized as follows. In Section 2, we derive the semiconductor QLEs and the noise correlations at the LIL. In Section 3, the semiconductor QLEs are

¹ The validity of this assumption with the meaning of the “cavity” will be explained in appendix A.

used to derive a new formula for the photon Fano factor. In Section 4, we examine the formula in various cases, and compare our theory with recent experiments [9–12]. In Section 5, we summarize the paper.

II. DERIVATION OF QUANTUM LANGEVIN EQUATIONS AND THE NOISE CORRELATIONS AT A LOW-INJECTION LEVEL

A. Microscopic Langevin equations of an LED

Chow *et al.* [21] discussed the case where a *single mode* is excited among many-photon modes of a cavity. Since we are mainly interested in the photon statistics of LEDs at the LIL, we extend their method to treat *many modes* of photons. The total Hamiltonian \mathcal{H}_{tot} (which describes multimode photons in the cavity and carriers in the active layer of an LED) is written as

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{multi-ph}} + \mathcal{H}_{\text{carrier}} + \mathcal{H}_{\text{dipole}} + \mathcal{H}_{\text{many-body}} + \mathcal{H}_{\text{baths}} + \mathcal{H}_{\text{baths-sys}}, \quad (1)$$

$$\mathcal{H}_{\text{multi-ph}} = \sum_l \hbar \nu_l a_l^\dagger a_l, \quad (2)$$

$$\mathcal{H}_{\text{carrier}} = \sum_{\mathbf{k}} \left[\left(\epsilon_g^0 + \frac{\hbar^2 k^2}{2m_e} \right) c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \frac{\hbar^2 k^2}{2m_h} d_{-\mathbf{k}}^\dagger d_{-\mathbf{k}} \right], \quad (3)$$

$$\mathcal{H}_{\text{dipole}} = \sum_{l,\mathbf{k}} \hbar (g_{l,\mathbf{k}}^0 d_{-\mathbf{k}}^\dagger c_{\mathbf{k}}^\dagger a_l + \text{h.c.}), \quad (4)$$

where $\mathcal{H}_{\text{multi-ph}}$ is the Hamiltonian of the multimode photons, a_l is the annihilation operator for the photons in mode l , and ν_l is the field oscillation frequency in mode l . The Hamiltonian of the electrons and holes in the active layer is $\mathcal{H}_{\text{carrier}}$. The annihilation operators of the electron and hole of wavevector \mathbf{k} are $c_{\mathbf{k}}$ and $d_{-\mathbf{k}}$, respectively, m_e and m_h are the electron and hole effective masses, respectively, and ϵ_g^0 is the bare band-gap energy. The interaction among the carriers and the photons is represented by $\mathcal{H}_{\text{dipole}}$ in the dipole approximation, with $g_{l,\mathbf{k}}^0$ being the bare coupling constants, and h.c. means Hermite conjugate. Note that we here consider only a direct radiative transition, however, in real devices, there are other processes, and we will include them when necessary. The many-body interaction between the carriers is represented by $\mathcal{H}_{\text{many-body}}$, the Hamiltonian of baths (or environments) by $\mathcal{H}_{\text{baths}}$, and the interaction between the baths and the system (the carriers and photons) by $\mathcal{H}_{\text{baths-sys}}$.

To obtain QLEs, we start with making a mean-field approximation for $\mathcal{H}_{\text{many-body}}$, and consequently ϵ_g^0 and $g_{\mathbf{k},l}^0$ are renormalized (See, *e.g.*, Chap. 4 of Ref. [21]). The renormalized parameters are denoted by ϵ_g and $g_{l,\mathbf{k}}$.

We then eliminate $\mathcal{H}_{\text{baths}} + \mathcal{H}_{\text{baths-sys}}$ by using the Markov approximation [3,16–21]. As a result, the fluctuation and dissipation terms appear in the equations of motion. The generalized Einstein relation [16–18,20,21]

gives the relation between the fluctuation and dissipation terms as follows,

$$2D_{\mu\nu} = \frac{d}{dt} \langle A_\mu A_\nu \rangle - \langle D_\mu A_\nu \rangle - \langle A_\mu D_\nu \rangle, \quad (5)$$

where $2D_{\mu\nu}$ is a diffusion coefficient, A_μ a system variable, D_μ a dissipation term in Langevin equations, i.e., $\dot{A}_\mu = D_\mu + F_\mu$, $\langle F_\mu(t) F_\nu(t') \rangle = 2D_{\mu\nu} \delta(t-t')$. The brackets mean the ensemble average for fluctuations.

We finally obtain the following microscopic QLEs, which describe LEDs in a microscopic scale, for the dipole operator $\sigma_{\mathbf{k}} = d_{-\mathbf{k}} c_{\mathbf{k}} e^{i\nu_l t}$, for the electric field operator $A_l(t) = a_l(t) e^{i\nu_l t}$, and for the electron occupation probability in \mathbf{k} space $n_{\mathbf{e}\mathbf{k}} = c_{\mathbf{k}}^\dagger c_{\mathbf{k}}$,

$$\begin{aligned} \frac{d}{dt} \sigma_{\mathbf{k}} = & -(\gamma + i\omega_{\mathbf{k}} - i\nu_l) \sigma_{\mathbf{k}} \\ & + i \sum_{l'} g_{l',\mathbf{k}} A_{l'} (n_{\mathbf{e}\mathbf{k}} + n_{\mathbf{h}\mathbf{k}} - 1) + F_{\sigma\mathbf{k}}, \end{aligned} \quad (6)$$

$$\frac{d}{dt} A_l = -\left[\frac{\kappa_l^0}{2} + i(\Omega_l - \nu_l)\right] A_l - i \sum_{\mathbf{k}} g_{l,\mathbf{k}}^* \sigma_{\mathbf{k}} + F_l, \quad (7)$$

$$\begin{aligned} \frac{d}{dt} n_{\mathbf{e}\mathbf{k}} = & P_{\mathbf{e}\mathbf{k}} (1 - n_{\mathbf{e}\mathbf{k}}) - \frac{n_{\mathbf{e}\mathbf{k}}}{\tau_{\text{nr}}} \\ & + \sum_l (i g_{l,\mathbf{k}}^* A_l^\dagger \sigma_{\mathbf{k}} + \text{h.c.}) + F_{\mathbf{e}\mathbf{k}}, \end{aligned} \quad (8)$$

where γ is the dipole decay (dephasing) rate, $\hbar\omega_{\mathbf{k}} \equiv \epsilon_g + \hbar^2 k^2 / 2m_e + \hbar^2 k^2 / 2m_h$ is the transition energy, and $F_{\sigma\mathbf{k}}$ is the fluctuation operator for the dipole. The hole occupation probability in \mathbf{k} space is $n_{\mathbf{h}\mathbf{k}} \equiv d_{-\mathbf{k}}^\dagger d_{-\mathbf{k}}$. The photon escape rate from the cavity is $\kappa_l^0 = \nu_l / Q_l$ where Q_l is the Q factor of the cavity, Ω_l is the passive-cavity frequency, and F_l is the fluctuation operator for the electric field. The pump rate due to a current injection or optical pumping is $P_{\mathbf{e}\mathbf{k}} (1 - n_{\mathbf{e}\mathbf{k}})$ where the factor $(1 - n_{\mathbf{e}\mathbf{k}})$ represents the pump blocking [21]. Note that a lifetime of non-radiative decay τ_{nr} has been introduced in Eq. (8). As discussed later, the existence of non-radiative processes is, in our model, a necessary condition for the difference between the quantum efficiency and the differential quantum efficiency to occur. The other condition is that the lifetime of radiative processes or that of non-radiative ones varies with n_e . [See Eqs. (66)-(68).] Non-radiative processes might be modelled by capture of carriers at a trapping level in an LED, however, we only need that τ_{nr} is an implicit function of n_e . These terms of pump and non-radiative decay have been phenomenologically introduced. The fluctuation operator for the electron number is $F_{\mathbf{e}\mathbf{k}}$.

B. Adiabatic approximation

To derive more useful forms for later discussion, we use the adiabatic approximation [19,23], and approximate the solution of Eq. (6) by

$$\sigma_{\mathbf{k}} \simeq \frac{i \sum_{l'} g_{l',\mathbf{k}} A_{l'} (n_{\mathbf{ek}} + n_{\mathbf{hk}} - 1) + F_{\sigma\mathbf{k}}}{\gamma + i\omega_{\mathbf{k}} - i\nu_l}, \quad (9)$$

Substituting Eq. (9) into (7), we find

$$\begin{aligned} \dot{A}_l &= -[\kappa_l^0/2 + i(\Omega_l - \nu_l)]A_l \\ &+ \sum_{l'} G_{ll'} A_{l'} + F_l + F_{\sigma,l}, \end{aligned} \quad (10)$$

where $G_{ll'}$ is a “gain matrix”;

$$G_{ll'} \equiv \sum_{\mathbf{k}} g_{l,\mathbf{k}}^* g_{l',\mathbf{k}} \mathcal{D}_{l,\mathbf{k}} (n_{\mathbf{ek}} + n_{\mathbf{hk}} - 1), \quad (11)$$

where $\mathcal{D}_{l,\mathbf{k}}$ is a complex Lorentzian; $\mathcal{D}_{l,\mathbf{k}} \equiv \frac{1}{\gamma + i(\omega_{\mathbf{k}} - \nu_l)}$. A new fluctuation operator $F_{\sigma,l}(t)$ has been defined by

$$F_{\sigma,l} \equiv -i \sum_{\mathbf{k}} g_{l,\mathbf{k}}^* \mathcal{D}_{l,\mathbf{k}} F_{\sigma\mathbf{k}}, \quad (12)$$

which is associated with the coupling between the carriers and photons.

From Eq. (10), we also find the QLE for the photon-number operator $n_l \equiv A_l^\dagger A_l$,

$$\begin{aligned} \frac{d}{dt} n_l &= -\kappa_l^0 n_l + \sum_{l'} [G_{ll'} A_l^\dagger A_{l'} + \text{h.c.}] \\ &+ [(F_{\sigma,l}^\dagger + F_l^\dagger) A_l + \text{h.c.}], \end{aligned} \quad (13)$$

Substituting Eq. (9) into (8), we also find the QLE for the total electron number $n_c \equiv \sum_{\mathbf{k}} n_{\mathbf{ek}}$,

$$\begin{aligned} \frac{d}{dt} n_c &= \sum_{\mathbf{k}} P_{\mathbf{ek}} (1 - n_{\mathbf{ek}}) - \frac{n_c}{\tau_{\text{nr}}} \\ &- \sum_{l,l'} [G_{ll'} A_l^\dagger A_{l'} + \text{h.c.}] \\ &+ \sum_{\mathbf{k}} F_{\mathbf{ek}} - \sum_l [A_l^\dagger F_{\sigma,l} + \text{h.c.}], \end{aligned} \quad (14)$$

C. Microscopic noise correlations of an LED

To discuss the statistical properties of light emitted from LEDs, we must determine the noise correlations. Hereafter we assume that the correlations between different modes of the photons and those between different wavenumbers of carriers can be neglected [19], *i.e.*,

$$\langle A_l^\dagger(t) A_{l'}(t) \rangle \simeq \langle n_l \rangle \delta_{ll'}, \quad (15)$$

$$\langle \sigma_{\mathbf{k}}^\dagger(t) \sigma_{\mathbf{k}'}(t) \rangle \simeq \langle n_{\mathbf{ek}} n_{\mathbf{hk}} \rangle \delta_{\mathbf{k},\mathbf{k}'}, \quad (16)$$

$$\langle \sigma_{\mathbf{k}}(t) \sigma_{\mathbf{k}'}^\dagger(t) \rangle \simeq \langle (1 - n_{\mathbf{ek}})(1 - n_{\mathbf{hk}}) \rangle \delta_{\mathbf{k},\mathbf{k}'}, \quad (17)$$

$$\langle n_{\mathbf{ek}}(t) n_{\mathbf{ek}'}(t) \rangle \simeq \langle n_{\mathbf{ek}} \rangle \delta_{\mathbf{k},\mathbf{k}'}. \quad (18)$$

We can then calculate the noise correlations, which are consistent with the QLEs (6)-(8), using (5);

$$\langle F_{\sigma\mathbf{k}}^\dagger(t) F_{\sigma\mathbf{k}'}(t') \rangle \simeq 2\gamma \langle n_{\mathbf{ek}} n_{\mathbf{hk}} \rangle \delta_{\mathbf{k},\mathbf{k}'} \delta(t - t'), \quad (19)$$

$$\langle F_{\sigma\mathbf{k}}(t) F_{\sigma\mathbf{k}'}^\dagger(t') \rangle \simeq 2\gamma \langle (1 - n_{\mathbf{ek}})(1 - n_{\mathbf{hk}}) \rangle \delta_{\mathbf{k},\mathbf{k}'} \delta(t - t'), \quad (20)$$

$$\langle F_l^\dagger(t) F_{l'}(t') \rangle \simeq \kappa_l^0 \bar{n}(\nu_l) \delta_{ll'} \delta(t - t'), \quad (21)$$

$$\langle F_l(t) F_{l'}^\dagger(t') \rangle \simeq \kappa_l^0 [\bar{n}(\nu_l) + 1] \delta_{ll'} \delta(t - t'), \quad (22)$$

$$\langle F_{\mathbf{ek}}(t) F_{\mathbf{ek}'}(t') \rangle = [\langle P_{\mathbf{ek}}(1 - n_{\mathbf{ek}}) \rangle + \langle n_{\mathbf{ek}}/\tau_{\text{nr}} \rangle] \delta_{\mathbf{k},\mathbf{k}'} \delta(t - t'), \quad (23)$$

where $\bar{n}(\nu_l)$ is the number of thermal photons in mode l in the cavity. We have used, in Eqs. (19) and (20), the quasiequilibrium conditions [21],

$$2\gamma \langle n_{\mathbf{ek}} n_{\mathbf{hk}} \rangle \gg \frac{d}{dt} \langle n_{\mathbf{ek}} n_{\mathbf{hk}} \rangle, \quad (24)$$

$$2\gamma \langle (1 - n_{\mathbf{ek}})(1 - n_{\mathbf{hk}}) \rangle \gg \frac{d}{dt} \langle (1 - n_{\mathbf{ek}})(1 - n_{\mathbf{hk}}) \rangle. \quad (25)$$

We can also calculate the correlations of $F_{\sigma,l}$;

$$\langle F_{\sigma,l}^\dagger(t) F_{\sigma,l}(t') \rangle = \langle R_{\text{sp},l} \rangle \delta(t - t'), \quad (26)$$

$$\langle F_{\sigma,l}(t) F_{\sigma,l}^\dagger(t') \rangle = \langle R_{\text{abs},l} \rangle \delta(t - t'), \quad (27)$$

where $R_{\text{sp},l}$ ($R_{\text{abs},l}$) denotes the spontaneous emission rate into photon mode l (the absorption rate of photon mode l);

$$R_{\text{sp},l} \equiv \frac{n_c}{\tau_{\text{r},l}} \equiv \frac{2}{\gamma} \sum_{\mathbf{k}} |g_{l,\mathbf{k}}|^2 \mathcal{L}_{l,\mathbf{k}} n_{\mathbf{ek}} n_{\mathbf{hk}}, \quad (28)$$

$$R_{\text{abs},l} \equiv \frac{2}{\gamma} \sum_{\mathbf{k}} |g_{l,\mathbf{k}}|^2 \mathcal{L}_{l,\mathbf{k}} (1 - n_{\mathbf{ek}})(1 - n_{\mathbf{hk}}). \quad (29)$$

We have used Eqs. (19) and (20), and defined $\tau_{\text{r},l}$ as a radiative lifetime of the carriers into mode l . The (dimensionless) Lorentzian line-shape function $\mathcal{L}_{l,\mathbf{k}}$ is $\mathcal{L}_{l,\mathbf{k}} \equiv \frac{\gamma^2}{\gamma^2 + (\omega_{\mathbf{k}} - \nu_l)^2}$.

At this stage, we have the following QLEs,

$$\frac{d}{dt} n_l = -\kappa_l n_l + [(F_{\sigma,l}^\dagger + F_l^\dagger) A_l + \text{h.c.}], \quad (30)$$

$$\begin{aligned} \frac{d}{dt} n_c &= \sum_{\mathbf{k}} P_{\mathbf{ek}} (1 - n_{\mathbf{ek}}) - \frac{n_c}{\tau_{\text{nr}}} \\ &- \sum_l (R_{\text{sp},l} - R_{\text{abs},l}) n_l \\ &+ \sum_{\mathbf{k}} F_{\mathbf{ek}} - \sum_l [A_l^\dagger F_{\sigma,l} + \text{h.c.}], \end{aligned} \quad (31)$$

where we have used

$$G_{ll} + G_{ll}^* = R_{\text{sp},l} - R_{\text{abs},l}, \quad (32)$$

and defined the renormalized photon escape rate

$$\kappa_l = \kappa_l^0 - R_{\text{sp},l} + R_{\text{abs},l}. \quad (33)$$

D. Semiconductor Langevin equations and the noise correlations for an LED at a low-injection level

In what follows, we assume ²

$$R_{\text{sp},l}, R_{\text{abs},l} \ll \kappa_l^0. \quad (34)$$

From Eqs. (30) and (34), the QLE for n_l at the LIL becomes

$$\frac{d}{dt}n_l = -\kappa_l^0 n_l + R_{\text{sp},l} + F_{n,l}, \quad (35)$$

where we have defined a new fluctuation operator;

$$F_{n,l} \equiv [(F_{\sigma,l}^\dagger + F_l^\dagger)A_l + \text{h.c.}] - (\kappa_l^0 \bar{n}(\nu_l) + R_{\text{sp},l}). \quad (36)$$

The right-hand side is averaged to be zero because the noises, $F_{n,l}$ and $F_{\sigma,l}$, have the Markovian property: $\langle F_l^\dagger A_l + \text{h.c.} \rangle = \kappa_l^0 \bar{n}(\nu_l)$ and $\langle F_{\sigma,l}^\dagger A_l + \text{h.c.} \rangle = R_{\text{sp},l}$.

From Eqs. (34) and (35), the steady-state value of the photon number becomes

$$(n_l)_{\text{s.s.}} \simeq R_{\text{sp},l}/\kappa_l^0 \ll 1, \quad (37)$$

i.e., the photon number in each mode is quite small.

On the other hand, from Eqs. (31), (34), and (37), the QLE for n_c at the LIL becomes

$$\frac{d}{dt}n_c = P - \frac{n_c}{\tau_r} - \frac{n_c}{\tau_{\text{nr}}} + F_c, \quad (38)$$

where $P \equiv \sum_{\mathbf{k}} P_{e\mathbf{k}}(1 - n_{e\mathbf{k}})$ is the total pump rate, and

$$\frac{1}{\tau_r} \equiv \frac{\sum_l R_{\text{sp},l}}{n_c} \equiv \sum_l \frac{1}{\tau_{r,l}}, \quad (39)$$

is the radiative decay rate (τ_r is the radiative lifetime of carriers). The fluctuation operator for the total electron number F_c is denoted by,

$$F_c \equiv \sum_l \left[-(A_l^\dagger F_{\sigma,l} + \text{h.c.}) + R_{\text{sp},l} \right] + \sum_{\mathbf{k}} F_{e\mathbf{k}}. \quad (40)$$

Note that the lifetimes ($\tau_{r,l}$ and τ_r) are carrier-number n_c dependent because $R_{\text{sp},l}$ is an implicit function of n_c .

We thus obtain the final forms of the semiconductor QLEs at the LIL as follows,

$$\frac{d}{dt}n_c = P - \frac{n_c}{\tau_r} - \frac{n_c}{\tau_{\text{nr}}} + \Gamma_P + \Gamma_r + \Gamma_{\text{nr}}, \quad (41)$$

$$\frac{d}{dt}n_l = -\kappa_l^0 n_l + \frac{n_c}{\tau_{r,l}} + F_{\kappa,l} + F_{r,l}, \quad (42)$$

where the noise operators, $F_{n,l}, F_c$, have been divided into new ones, $\Gamma_r, F_{r,l}, \Gamma_{\text{nr}}, \Gamma_P, F_{\kappa,l}$. They are given by

$$\Gamma_r \equiv \sum_l \left[-(A_l^\dagger F_{\sigma,l} + \text{h.c.}) + R_{\text{sp},l} \right], \quad (43)$$

$$F_{r,l} \equiv A_l^\dagger F_{\sigma,l} + \text{h.c.} - R_{\text{sp},l}, \quad (44)$$

$$\Gamma_{\text{nr}} + \Gamma_P \equiv \sum_{\mathbf{k}} F_{e\mathbf{k}}, \quad (45)$$

$$F_{\kappa,l} \simeq A_l^\dagger F_l + F_l^\dagger A_l, \quad (46)$$

where we have neglected the number of thermal photons, $\bar{n}(\nu_l)$, since this is the case in usual experiments. These noise operators ($\Gamma_r, F_{r,l}, \Gamma_{\text{nr}}, \Gamma_P, F_{\kappa,l}$) are associated with the radiative decay of carriers, the conversion from carriers to photons, the non-radiative decay of carriers, the (intrinsic) pump fluctuation, and the photon escape from the cavity, respectively. The mean values of the noises are defined to be zero, of course.

From above the noise correlation for Γ_r can be calculated as,

$$\begin{aligned} & \langle \Gamma_r(t) \Gamma_r(t') \rangle \\ & \simeq \sum_l \left[\langle A_{c,l}^\dagger(t) A_{c,l}(t') \rangle \langle F_{\sigma,l}(t) F_{\sigma,l}^\dagger(t') \rangle \right. \\ & \quad \left. + \langle A_{c,l}(t) A_{c,l}^\dagger(t') \rangle \langle F_{\sigma,l}^\dagger(t) F_{\sigma,l}(t') \rangle \right] \\ & = \sum_l [\langle R_{\text{sp},l} + R_{\text{abs},l} \rangle \langle n_l \rangle + \langle R_{\text{sp},l} \rangle] \delta(t - t') \\ & \simeq \sum_l \langle R_{\text{sp},l} \rangle \delta(t - t') = \left\langle \frac{n_c}{\tau_r} \right\rangle \delta(t - t'), \end{aligned} \quad (47)$$

where, as done in Eq. (15), we have neglected, in the second and third lines, the correlations between different modes. We also have assumed the following,

$$\Gamma_r \simeq \sum_l \left[-(A_{c,l}^\dagger F_{\sigma,l} + \text{h.c.}) \right],$$

where $A_{c,l}^\dagger$ is an operator uncorrelated with $F_{\sigma,l}$ [24]. Thus the property of fluctuations, $\langle \Gamma_r \rangle = 0$, still holds. Equations (26) and (27) have also been used in the fourth line of Eq. (47), and Eqs. (34), (37), and (39) in the fifth line of Eq. (47).

Similarly we obtain

$$\langle \Gamma_r(t) F_{r,l}(t') \rangle = - \left\langle \frac{n_c}{\tau_{r,l}} \right\rangle \delta(t - t'), \quad (48)$$

$$\langle F_{r,l}(t) F_{r,l'}(t') \rangle = \left\langle \frac{n_c}{\tau_{r,l}} \right\rangle \delta_{ll'} \delta(t - t'), \quad (49)$$

$$\langle \Gamma_{\text{nr}}(t) \Gamma_{\text{nr}}(t') \rangle = \left\langle \frac{n_c}{\tau_{\text{nr}}} \right\rangle \delta(t - t'), \quad (50)$$

$$\langle \Gamma_P(t) \Gamma_P(t') \rangle = \langle P \rangle \delta(t - t'), \quad (51)$$

where we have divided the correlation $\langle \Gamma_{\text{nr}}(t) \Gamma_{\text{nr}}(t') \rangle + \langle \Gamma_P(t) \Gamma_P(t') \rangle$ into two parts; one is the correlation of the

² The following inequality, at first sight, might look rather strange. We therefore explain its validity *at the LIL* in the appendix A.

non-radiative processes, and the other that of the pump. These semiconductor QLEs, (41) and (42), and the noise correlations (47)-(51) were just assumed by us [22,25] and Yamanishi and Lee (YL) [5], although YL did not include non-radiative processes.

Furthermore, we focus on the total photon flux, N , which can be detected at the PD surface. To this end, we must find two relations: one is the relation between the photon number in the cavity and the photon flux from the cavity. The other is the relation between the photon flux from the cavity and the photon flux detected at the PD surface. The input-output formalism [3,20] gives the first relation as,

$$V_l = \kappa_l^0 n_l - F_{\kappa,l}, \quad (52)$$

where V_l is the photon flux of mode l from the cavity. We next model the second relation using the argument of a beam splitter [20]. The relation between the mean flux of N and that of V_l is given by

$$\langle N \rangle = \sum_l \xi_l \langle V_l \rangle \equiv \beta_0 \sum_l \langle V_l \rangle \equiv \beta_0 \langle V \rangle, \quad (53)$$

where ξ_l is a “transmission coefficient” of mode l (see Fig. 1),

FIG. 1. Definition of the ξ_l . Photons of mode l are emitted into the region between the solid lines. The photodetector can detect only some region (denoted by the one between the dashed lines) of the photons because its surface area is limited and the quantum efficiency η of the detector is less than one. ξ_l is the probability that a photon of mode l is detected by the detector. It increases as the surface area and/or η do.

β_0 is the transfer efficiency [5], and $V \equiv \sum_l V_l$ is the total photon flux from the cavity. The relation between the flux correlation of N and that of V_l is given by

$$\begin{aligned} \langle |\Delta \tilde{N}(\Omega)|^2 \rangle &= \sum_l \xi_l (1 - \xi_l) \langle \tilde{V}_l(\Omega) \rangle \\ &+ \sum_{l,l'} \xi_l \xi_{l'} \langle \Delta \tilde{V}_l(\Omega) \Delta \tilde{V}_{l'}(\Omega) \rangle, \end{aligned} \quad (54)$$

where $\tilde{N}(\Omega)$ and $\tilde{V}_l(\Omega)$ denote Fourier components of N and V_l , respectively.

III. CALCULATION OF THE PHOTON FANO FACTOR

In this section, we calculate the Fano factor of photons [6] which denotes the normalized fluctuation of the photon number detected at the PD surface.

Following the standard small-signal analysis used in Refs. [4,5,7,14], we expand the radiative and non-radiative lifetimes, $\tau_{r,l}[n_c]$, $\tau_r[n_c]$, and $\tau_{nr}[n_c]$, to linear order in $\Delta n_c \equiv n_c - n_{c0}$;

$$\tau_{r,l}[n_c] = (\tau_{r,l})_0 (1 - K_{r,l} \frac{\Delta n_c}{n_{c0}}), \quad (55)$$

$$\tau_r[n_c] = \tau_{r0} (1 - K_r \frac{\Delta n_c}{n_{c0}}), \quad (56)$$

$$\tau_{nr}[n_c] = \tau_{nr0} (1 + K_{nr} \frac{\Delta n_c}{n_{c0}}), \quad (57)$$

$$\frac{1}{\tau_{r0}} \equiv \sum_l \frac{1}{(\tau_{r,l})_0}, \quad (58)$$

$$K_r = \sum_l \frac{\tau_{r0}}{(\tau_{r,l})_0} K_{r,l}, \quad (59)$$

where quantities with the subscript 0 denote the average values at a running point, $P = P_0$. Sensitivity of lifetimes to the carrier-number fluctuations Δn_c is represented by $K_{r,l}$, K_r and K_{nr} . Though K_r is usually positive, it is not always the case as we will discuss in the next section.

Linearizing Eqs. (41), (42) and (52) in terms of Δn_c , $\Delta n_l \equiv n_l - (n_l)_0$, $\Delta P \equiv P - P_0$, and $\Delta V_l \equiv V_l - (V_l)_0$, and substituting Eqs. (55) and (57) into them, we obtain

$$\frac{d}{dt} \Delta n_c = \Delta P - \frac{\Delta n_c}{\tau''} + \Gamma_P + \Gamma_r + \Gamma_{nr}, \quad (60)$$

$$\frac{d}{dt} \Delta n_l = -\kappa_l^0 \Delta n_l + \frac{\Delta n_c}{\tau_{r,l}} + F_{\kappa,l} + F_{r,l}, \quad (61)$$

$$\Delta V_l = \kappa_l^0 \Delta n_l - F_{\kappa,l}, \quad (62)$$

where we have used the following equilibrium conditions:

$$\begin{aligned} P_0 &= n_{c0} \left(\frac{1}{\tau_{r0}} + \frac{1}{\tau_{nr0}} \right), \\ \sum_l \kappa_l^0 (n_l)_0 &= \sum_l \frac{n_{c0}}{(\tau_{r,l})_0} = \frac{n_{c0}}{\tau_{r0}} \\ &= \sum_l V_{l0} = V_0 = \frac{P_0}{1 + (\tau_{r0}/\tau_{nr0})}, \end{aligned} \quad (63)$$

and introduced the effective lifetimes defined as

$$\tau'_r \equiv \frac{\tau_{r0}}{1 + K_r}, \quad \tau'_{r,l} \equiv \frac{(\tau_{r,l})_0}{1 + K_{r,l}}, \quad \tau'_{nr} \equiv \frac{\tau_{nr0}}{1 - K_{nr}}, \quad (64)$$

$$\frac{1}{\tau''} \equiv \frac{1}{\tau'_r} + \frac{1}{\tau'_{nr}}. \quad (65)$$

Dropping the noise terms in Eqs. (60)-(62), and using Eq. (53), we calculate the quantum efficiency η and the differential quantum efficiency η_d as

$$\eta \equiv \frac{N_0}{P_0} = \frac{\beta_0/\tau_{r0}}{(1/\tau_{r0} + 1/\tau_{nr0})} = \frac{\beta_0}{1 + \epsilon_0}, \quad (66)$$

$$\eta_d \equiv \frac{\Delta \tilde{N}(\Omega)}{\Delta \tilde{P}(\Omega)} \bigg|_{\Omega \rightarrow 0} = \frac{\beta_0/\tau'_r}{(1/\tau'_r + 1/\tau'_{nr})} = \frac{\beta_0}{1 + \epsilon'}, \quad (67)$$

where

$$\epsilon_0 \equiv \frac{\tau_{r0}}{\tau_{nr0}}, \quad \epsilon' \equiv \frac{\tau'_r}{\tau'_{nr}} = \frac{1 - K_{nr}}{1 + K_r} \epsilon_0. \quad (68)$$

These efficiencies (η and η_d) are illustrated in Fig. 2.

FIG. 2. Typical I-L (Injection-Light) characteristics of LEDs are shown by the solid lines. The quantum efficiency η is the slope of the straight line (represented by the dashed line) connecting the origin and the running point, whereas the differential quantum efficiency η_d is the slope of the tangential line (chain line) at the running point.

Note that the difference between η and η_d , which was measured to be large (*e.g.*, $\eta_d/\eta > 2$ at the LIL [12]), can not be explained by the previous theories which assumed $\tau_{nr0} = \infty$ [4–6] or $K_r = K_{nr} = 0$ [7]. In this sense, our model is a *minimal* one that simulates real LEDs.

Hereafter we assume that $\Omega \ll \kappa_l^0$ ($\sim 10^{12}$ Hz), which is well satisfied in usual experimental conditions. Then the response of the output photon flux to the modulation of the pump is calculated as,

$$\left| \frac{\Delta \tilde{N}(\Omega)}{\Delta \tilde{P}(\Omega)} \right| = \frac{\eta_d}{\sqrt{1 + \Omega^2 \tau''^2}}. \quad (69)$$

Hence $1/\tau''$ is understood as a cutoff frequency. It is seen that the non-radiative processes push the cutoff to the higher frequency [See Eq. (65)].

On the other hand, the photon flux correlation between mode l and m is calculated using Eqs. (60)-(62) as

$$\begin{aligned} & \langle \Delta \tilde{V}_l^*(\Omega) \Delta \tilde{V}_m(\Omega) \rangle \\ &= \frac{\tau''}{\tau'_{r,l}} \frac{\tau''}{\tau'_{r,m}} \frac{\langle |\Delta \tilde{P}_{\text{tot}}|^2 \rangle + \langle |\tilde{\Gamma}_r|^2 \rangle + \langle |\tilde{\Gamma}_{nr}|^2 \rangle}{1 + (\Omega \tau'')^2} \\ &+ \frac{\tau''}{\tau'_{r,l}} \frac{\langle \tilde{\Gamma}_r^* \tilde{F}_{r,m} \rangle}{1 - i\Omega \tau''} + \frac{\tau''}{\tau'_{r,m}} \frac{\langle \tilde{\Gamma}_r^* \tilde{F}_{r,l} \rangle}{1 + i\Omega \tau''} + \langle \tilde{F}_{r,l}^* \tilde{F}_{r,m} \rangle, \end{aligned} \quad (70)$$

where a total pump noise $\Delta \tilde{P}_{\text{tot}}$ has been defined as

$$\Delta \tilde{P}_{\text{tot}}(\Omega) \equiv \Delta \tilde{P}(\Omega) + \tilde{\Gamma}_P(\Omega),$$

which consists of the modulation $\Delta \tilde{P}(\Omega)$ and the intrinsic pump noise $\tilde{\Gamma}_P(\Omega)$. Note that the latter noise can be suppressed by inelastic scattering in conductors [4,8–10,26].

To see the physical meaning of Eq. (70), let us introduce the Fano factor of the pump electrons (or excitons) W_e and that of the photons detected at the PD surface W_{ph} , which are defined by

$$W_e(\Omega) \equiv \frac{\langle |\Delta \tilde{P}_{\text{tot}}(\Omega)|^2 \rangle}{P_0 T}, \quad (71)$$

$$W_{ph}(\Omega) \equiv \frac{\langle |\Delta \tilde{N}(\Omega)|^2 \rangle}{N_0 T}, \quad (72)$$

where T is the Fourier-integral time. We transform Eqs. (47)-(51) into the Fourier components:

$$\begin{aligned} & \langle \tilde{F}_r^*(\Omega) \tilde{F}_r(\Omega) \rangle = \langle \tilde{\Gamma}_r^*(\Omega) \tilde{\Gamma}_r(\Omega) \rangle = \frac{1}{\epsilon_0} \langle \tilde{\Gamma}_{nr}^*(\Omega) \tilde{\Gamma}_{nr}(\Omega) \rangle \\ &= -\langle \tilde{\Gamma}_r^*(\Omega) \tilde{F}_r(\Omega) \rangle = \frac{n_{c0}}{\tau_{r0}} T = V_0 T = \frac{P_0 T}{1 + \epsilon_0}. \end{aligned} \quad (73)$$

Substituting Eqs. (70) and (73) into (54), and dividing by $N_0 T$, we finally obtain

$$W_{ph}(\Omega) = 1 - \frac{2\eta_d \zeta_1}{1 + (\Omega \tau'')^2} + \frac{\eta_d^2}{\eta} \frac{1 + W_e(\Omega)}{1 + (\Omega \tau'')^2} \zeta_2, \quad (74)$$

where we have used Eqs. (53), (63), (66)-(68), and defined the following,

$$\zeta_1 \equiv \sum_{l,m} \frac{\tau'_r}{\tau'_{r,l}} \frac{\tau_{r0}}{(\tau_{r,m})_0} \frac{\xi_l}{\beta_0} \frac{\xi_m}{\beta_0}, \quad (75)$$

$$\zeta_2 \equiv \left[\sum_l \frac{\tau'_r}{\tau'_{r,l}} \frac{\xi_l}{\beta_0} \right]^2, \quad (76)$$

which represent effects due to multimodeness of LEDs, and the consequences are discussed below. Note also that $\zeta_1 \leq 1, \zeta_2 \leq 1$.

Equation (74) is our main result which gives the Fano factor of the photons detected at the PD surface as a function of the Fano factor of the pump, measuring frequency, and several parameters ($\eta, \eta_d, \zeta_1, \zeta_2$, and τ''). Note that $1/\tau''$ becomes a cutoff frequency of the Fano factor as well as that of the modulation [see Eq. (69)].

IV. DISCUSSION

A. Low-frequency limit

Let us examine Eq. (74) in two cases, $\Omega = 0$ and $\Omega > 0$, separately. We first discuss the low-frequency limit. This case is applicable to the most experiments, because they are usually performed at the frequency which is lower than any other relevant frequencies. Setting $\Omega \rightarrow 0$, we obtain

$$W_{ph}(0) = 1 - 2\eta_d \zeta_1 + \frac{\eta_d^2}{\eta} [1 + W_e(0)] \zeta_2. \quad (77)$$

There are several cases where this expression becomes simpler:

1. The case where photons are emitted and detected homogeneously (the homogeneous case)

(h-1): When photons in each mode are emitted and detected *homogeneously*, *i.e.*, when $K_{r,l} = K_r = \text{constant}$ and $\xi_l = \beta_0 = \text{constant}$, we have $\zeta_1 = \zeta_2 = 1$, hence

$$W_{ph}(0) = 1 - 2\eta_d + \frac{\eta_d^2}{\eta} [1 + W_e(0)]. \quad (78)$$

This is the formula which we have derived elsewhere [25]. We have implicitly assumed this property (h-1) there, and this is a natural choice unless one considers a situation such that inhomogeneity due to, *e.g.*, cavity-QED

effects becomes important.

(h-2): In addition to (h-1), when the non-radiative processes do not exist and/or the carrier-number dependence of lifetimes cancels to be zero, *i.e.*, $\tau_{nr0} \rightarrow \infty$ and/or $K_r + K_{nr} = 0$, we have $\eta = \eta_d$ and the I-L (Injection-Light) characteristics become straight. Hence, from Eq. (78), we obtain

$$\begin{aligned} W_{ph}(0) &= 1 - 2\eta + \eta[1 + W_e(0)] \\ &= 1 - \eta + \eta W_e(0). \end{aligned} \quad (79)$$

This is just the previous formula which is frequently used in the literature [6–9,13].

2. The case where photons are emitted and/or detected inhomogeneously (the inhomogeneous case)

Note that in the homogeneous case the factors ζ_1 and ζ_2 do not appear in the expressions for W_{ph} [Eqs. (78) and (79)]. On the other hand, they appear when emission and/or detection efficiencies are different among different modes. We call this general case the “inhomogeneous case.” To investigate this case, let us consider a simple case where $K_{r,l} = 0$ and $K_{nr} \neq 0$. In this case, $\zeta_1 = \zeta_2 \equiv \zeta$, where

$$\zeta \equiv \left[\sum_l \frac{\tau_{r0}}{(\tau_{r,l})_0} \frac{\xi_l}{\beta_0} \right]^2. \quad (80)$$

Hence, from Eq. (77), we obtain

$$W_{ph}(0) = 1 - 2\eta_d\zeta + \frac{\eta_d^2\zeta}{\eta}[1 + W_e(0)]. \quad (81)$$

Compared with Eq. (78), we see that η and η_d are effectively multiplied by ζ in Eq. (81). However, this does not mean that ζ could be absorbed in η and η_d , because they are already defined by Eqs. (66) and (67), respectively. Any redefinition would lead to disagreement with the observed I-L characteristics.

3. Condition for generation of sub-Poissonian light

As another illustration of our result, we next discuss the condition for generation of sub-Poissonian light (SPL) ($W_{ph} < 1$) with a Poissonian pump ($W_e = 1$). For simplicity, we hereafter consider the case (h-1) [Eq. (78)], because many LEDs seem to be categorized in this case. From Eq. (78), we obtain

$$0 < \eta_d < \eta, \quad (82)$$

as the condition. This can be intuitively understood that the flatter the I-L characteristics, the duller the sensitivity of the LED to the pump fluctuation. Hence, if I-L characteristics are like Fig. 2 (b), then *even a Poissonian*

pump ($W_e = 1$) *can produce sub-Poissonian light*. Note that this mechanism is completely different from those of Refs. [7,13,27], because the authors of [7,13,27] eventually make the current noise injected to the active layers *below Poissonian*. On the other hand, the condition (82) is equivalent to

$$K_r + K_{nr} < 0, \quad (83)$$

where we have used Eqs. (66)-(68). For simplicity, we here take $K_{nr} = 0$, and find below what is needed for $K_r < 0$.

The spontaneous emission (SE) rate n_c/τ_r can be expressed approximately as $n_c/\tau_r \propto (n_c)^p$, where p is a constant. We therefore have $K_r \simeq p - 1$ from Eq. (56). It is well known [21] that $p \simeq 1$ ($p \simeq 2$) in high- (low-) injection regions for SE processes of free carriers. For exciton recombination, we have $p \simeq 1$ at the LIL. Thus we usually have non-negative values of K_r . Making use of cavity-QED effects, however, we can obtain *negative* values of K_r . This is illustrated in Fig. 3 for a p -doped quantum well structure in a micro cavity, where we have assumed the following: the conduction band is parabolic, the effective electron mass is 0.1 times the free electron mass, the doping level is high, and the cavity-QED effects prohibit SE except at the band edge.

FIG. 3. The spontaneous emission (SE) rate as a function of carrier population in a doped-quantum-well structure in a micro cavity. Three curves correspond to different temperatures: $T = 80\text{K}$ (bottom), $T = 15\text{K}$ (middle), and $T = 3\text{K}$ (top).

It is seen that K_r becomes negative for the sheet carrier density $\gtrsim 10^{14}\text{m}^{-2}$ when temperatures are low enough (maybe below 3K). In this case, we obtain $\eta_d < \eta$ [see Eqs. (66)-(68)], and even a super-Poissonian pump can produce SPL. Exciton recombination with cavity-QED effects would work better, which will be discussed elsewhere.

It has been therefore shown that our formula (74) is useful to find the condition for generation of SPL, or, of course, other quantum states of light.

B. Finite-frequency cases

Let us turn to the finite frequency cases. For simplicity, we here use Eq. (74) with property (h-1), *i.e.*, $\zeta_1 = \zeta_2 = 1$, and assume the case where $K_r = K_{nr}$ in Figs. 4.

FIG. 4. The photon Fano factors are plotted as functions of frequency Ω (in units of $1/\tau_{r0}$). (a) and (b) represent the cases where $K_r = K_{nr} = 0.5$. (c) and (d) the cases where $K_r = K_{nr} = -0.5$. The pump noise level is assumed to be zero ($W_e = 0$) and the Poissonian level ($W_e = 1$) in (a), (c) and (b), (d), respectively. In each figure, the solid and dashed lines represent the cases where the non-radiative recombination is significant ($\tau_{r0}/\tau_{nr0} = 1$) and absent ($\tau_{r0}/\tau_{nr0} = 0$), respectively. The cutoff frequencies are indicated by vertical arrows.

The solid lines (dashed lines) in Figs. 4 represent the cases when non-radiative processes exist (do not exist).

1. When $K_r = K_{nr} = 0.5$

The noiseless ($W_e = 0$) and Poissonian ($W_e = 1$) pump are denoted by Figs. 4 (a) and (b), respectively. When $\tau_{nr0} = \infty$ (dashed lines), we recover the results similar to those of Yamanishi and Lee [5]. We also see that the cutoff frequency (indicated by vertical arrow) for $\tau_{nr0} \neq \infty$ becomes higher than that for $\tau_{nr0} = \infty$ [Fig. 4 (a)].

2. When $K_r = K_{nr} = -0.5$

The noiseless ($W_e = 0$) and Poissonian ($W_e = 1$) pump are denoted by Figs. 4 (c) and (d), respectively. We find that the existence of non-radiative processes gives smaller W_{ph} in some frequency region [Fig. 4 (c)] or in the entire frequency range [Fig. 4 (d)]. In particular, Fig. 4 (d) shows that the Poissonian pumping can produce SPL in a wide frequency range. Figure 4 (c) also shows that the cutoff frequency for $\tau_{nr0} \neq \infty$ becomes higher than that for $\tau_{nr0} = \infty$ as well as Fig. 4 (a) does.

It might be difficult, however, to observe these features experimentally, because they would not appear up to $1/\tau_{r0} \sim 1\text{GHz}$. (Recently the photon Fano factor of LEDs has been measured up to 40 MHz [12].)

C. Comparison with the experimental results

Hirano and Kuga [9] reported that the previous formula (79) disagrees with their experimental data in low-frequency regions: They measured the following ratio

$$\frac{W_{ph}(W_e = 0)}{W_{ph}(W_e = 1)} \equiv r, \quad (84)$$

and obtained that $r < 1 - \eta$, whereas Eq. (79) gives $r = 1 - \eta$. Since the measuring frequency Ω_{meas} is low enough, *i.e.*, $\Omega_{meas} \sim 10\text{MHz} \ll 1/\tau'' \sim 1\text{GHz}$, and their LEDs seem to be categorized in the homogeneous case (h-1), we can use Eq. (78) to analyze their results. Our formula (78) gives

$$r = \frac{1 - 2\eta_d + \eta_d^2/\eta}{1 - 2\eta_d + 2\eta_d^2/\eta}. \quad (85)$$

When $\eta_d > \eta$ (which is usually the case in low-injection regions [9–12]), our theory gives $r < 1 - \eta$, in agreement with the experimental results. Furthermore, Hirano *et al.* [11] recently confirmed that Eq. (85) agrees with their experimental data (see the table below). It is seen that the values of the previous theory ($r = 1 - \eta$) are larger than the experimental values, whereas the values of Eq. (85) are closer to them. It also has been shown that, when $W_e = 1$, W_{ph} itself is more than one [11]. This fact also supports our formula (78) because it gives, when $W_e = 1$ and $\eta_d > \eta$, $W_{ph} = 1 - 2\eta_d + 2\eta_d^2/\eta > 1$, and their LEDs also have the property $\eta_d > \eta$ [9,11].

On the other hand, a simple argument [11,28] leads to

$$W_{ph}(0) = 1 - \eta + \frac{\eta_d^2}{\eta} W_e(0). \quad (86)$$

Since $|\eta - \eta_d|$ is small in Table I, the difference between this formula and Eq. (78) is within the experimental error and is not detectable. Further experimental studies are needed to observe the difference.

V. SUMMARY

From the microscopic QLEs (6)-(8), the associated noise correlations (19)-(23), and the assumption (34), we have derived the effective semiconductor QLEs (41), (42) and the associated noise correlations (47)-(51). The assumption (34) is valid at a low-injection level and in real devices as explained in appendix A. Applying the semiconductor QLEs to semiconductor LEDs, we obtain a new formula (74) for the Fano factor of photons. It gives the photon-number statistics as a function of the pump statistics and several parameters of LEDs, which are defined by (65)-(68), (75) and (76). Key ingredients are non-radiative processes, carrier-number dependence of lifetimes, and multimodeness of LEDs. The formula is applicable to the actual cases where the quantum efficiency η differs from the differential quantum efficiency η_d , whereas the previous theories [4–7] turn out to give $\eta = \eta_d$. It is also applicable to cases where photons in each mode of the cavity are emitted and/or detected inhomogeneously. When $\eta_d < \eta$ at the running point, in particular, our formula predicts that even a Poissonian pump can produce sub-Poissonian light (see Sec. IV A 3). This mechanism for generation of sub-Poissonian light is completely different from those of the previous theories, which assumed sub-Poissonian statistics for the current injected into the active layers of LEDs. It was shown that our results agree with recent experiments by Hirano, Kuga and coworkers [9–12]. We have also found that, in finite frequency regions, non-radiative processes sometimes give better results (smaller W_{ph} and/or a higher cutoff frequency). These will deserve further theoretical

and experimental researches of quantum aspects of light emitted from LEDs.

VI. ACKNOWLEDGMENTS

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APPENDIX A: VALIDITY OF THE INEQUALITY

We here show that the inequality (34), $R_{\text{sp},l}, R_{\text{abs},l} \ll \kappa_l^0$, holds for LEDs at a low-injection level (LIL).

At the LIL, pumped carriers first relax to lower-energy states which are formed by impurities, defects, spatial randomness and so on, and then recombine to radiate photons [Fig. 5 (a)]. Within the energy region of our interest, where photons are emitted, we thus have $R_{\text{sp},l} \sim R_{\text{abs},l}$. In addition, in real devices, there exists a Stokes shift [Fig. 5 (b)] which makes, at low temperatures, $R_{\text{abs},l}$ even smaller. Hence we have the absorption and emission profiles such as Fig. 5 (c).

We now proceed to compare $R_{\text{sp(abs)},l}$ with κ_l^0 . In Eqs. (28) and (29), $|g_{l,\mathbf{k}}|^2$ is proportional to $1/V_{\text{cavity}}$ (V_{cavity} is the volume of a cavity), and $\sum_{\mathbf{k}}$ is to the volume of the active layer V_{active} , hence we have $R_{\text{sp(abs)},l} \propto V_{\text{active}}/V_{\text{cavity}}$. In the analysis of *laser diodes* (LDs), it is customary to take $V_{\text{active}} \simeq V_{\text{cavity}}$, because the devices are made so that the lasing modes are confined in the active layer. In this case, we usually have $R_{\text{abs},l}/c \sim 10^4 \text{cm}^{-1} \gg \kappa_l^0/c \sim 10^2 \text{cm}^{-1}$ (c is the velocity of light), thus we can not obtain the inequality (34). In the case of *LEDs*, on the other hand, they are usually designed in such a way that the reflection coefficients on the boundary surfaces are small, hence most modes of photons of our interest are not confined in the layer. In this case, it is natural to take the “cavity” volume as big as a cube on which the detector’s surface is located [Fig. 5 (d)]. Then κ_l^0 can be estimated as $1/\kappa_l^0 \simeq (V_{\text{cavity}})^{1/3}/c + Qt_{\text{device}}$, where t_{device} is a time for photons to traverse the device. Since Q and t_{device} are small for LEDs, we have $\kappa_l^0 \simeq c/(V_{\text{cavity}})^{1/3}$. Therefore, noting that $R_{\text{abs},l} \propto 1/V_{\text{cavity}}$ and $\kappa_l^0 \propto 1/(V_{\text{cavity}})^{1/3}$, we can have the relation $R_{\text{abs},l} \ll \kappa_l^0$, if we take V_{cavity} big enough. Thus we obtain the inequality (34) for the case of LEDs at the LIL.

FIG. 5. (a) A schematic band diagram of real LEDs at the LIL, (b) the Stokes shift [CB (VB) represents the conduction (valence) band.], (c) the absorption and emission profiles at the LIL, and (d) the active layer, the device (LED), and the “cavity”.

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TABLE I. The experimental values of η, η_d, r and the theoretical values of r . After Ref. [11].

$\eta(\text{exp.})$	$\eta_d(\text{exp.})$	$r(\text{exp.})$	$r(\text{Eq. (85)})$	$r = 1 - \eta$
0.067	0.090	0.90	0.89	0.93
0.104	0.125	0.84	0.86	0.90
0.150	0.175	0.81	0.81	0.85















